

PARTONS: PARtonic Tomography Of Nucleon Software

A computing platform for the phenomenology of Generalized Parton Distributions

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Abstract We describe the architecture and functionalities of a C++ software framework, coined PARTONS, dedicated to the phenomenology of Generalized Parton Distributions. PARTONS provides a necessary bridge between models of Generalized Parton Distributions and experimental data measured in various exclusive channels. We outline the specifications of the PARTONS project in terms of practical needs, physical content and numerical capacity. This framework will be useful not only for theorists to develop new models but also to interpret existing measurements and even design new experiments.

Keywords Quantum Chromodynamics · Nucleon structure · Quark · Gluon · Parton · Generalized Parton Distributions · Compton Form Factors · Deeply Virtual Compton Scattering · Jefferson Lab · COMPASS · Software · Service Oriented Architecture · Multilayered Architecture · C++ · XML · SQL

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Generalized Parton Distributions (GPDs) were discovered in 1994 by Müller *et al.* [1] and independently introduced in 1997 by Radyushkin [2] and Ji [3]. This subfield of QCD grew rapidly because of the unique theoretical, phenomenological and experimental properties of these objects. GPDs are related to other non-perturbative quantities that were studied previously without connection: Parton Distribution Functions (PDFs) and Form Factors (FFs). In an infinite-momentum frame, where a hadron is flying at near the speed of light, PDFs describe the longitudinal momentum distribu-

tions of partons inside the hadron and FFs are the Fourier transforms of the hadron charge distribution in the transverse plane. PDFs and FFs appear as limiting cases of GPDs and in the pion case GPDs also extend the notion of a Distribution Amplitude (DA). This generality is complemented by one remarkable feature: GPDs are directly connected to the matrix elements of the QCD energy-momentum tensor evaluated between hadron states. This is both welcome and unexpected because the energy-momentum tensor in canonically probed through gravity. GPDs bring the energy-momentum matrix elements within experimental reach through electromagnetic scattering. It was realized from the early days that the electroproduction of a real photon off a nucleon target in the spacelike region, referred to as Deeply Virtual Compton Scattering (DVCS), is the theoretically cleanest way to access GPDs. At the beginning of the 21st century, first measurements of DVCS were reported by the HERMES [4] and CLAS [5] collaborations, establishing the immediate experimental relevance of the concept and marking the beginning of the experimental era of this field. Several dedicated experiments and sophisticated theoretical developments followed, putting the field in a good shape as many reviews testify [6–12].

GPDs are natural extensions of PDFs and yet their phenomenology is much harder. The lack of a general first principles parameterization justifies the need for several models, while the large number of possibly involved functions requires a multichannel analysis to constrain GPDs from different experimental filters. GPDs belong to an active research field where deep theoretical questions are to be solved, in conjunction with existing experimental programs, technological challenges, computational issues, and well-defined objects and mea-

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surements. The foreseen accuracy of experimental data to be measured at Jefferson Lab or at COMPASS requires the careful design of tools to meet the challenge of the high-precision era, and to be able to make the best from experimental data. The same tools should also be used for the design of future experiments. This is the aim of the PARTONS code.

The first section is a reminder of the phenomenological framework: how GPDs are defined, and how they can be experimentally access. We will illustrate the discussion with the example of DVCS. Then we discuss the need assessments for high precision GPD phenomenology in the second section, and our development environment in the third section. The fourth section describes the code architecture, while the fifth section lists existing modules. The sixth and last section provides several examples.

1 Phenomenological framework

1.1 Definition of Generalized Parton Distributions

In the unpolarized (vector) sector, spin-1/2 GPDs are defined in the light cone gauge from the following matrix elements:

$$F^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle P + \frac{\Delta}{2} \left| \bar{q} \left(-\frac{z}{2} \right) \gamma^+ q \left(\frac{z}{2} \right) \right| P - \frac{\Delta}{2} \right\rangle \Big|_{\substack{z^+=0 \\ z_\perp=0}}, \quad (1)$$

$$F^g(x, \xi, t) = \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle P + \frac{\Delta}{2} \left| G_a^{+\mu} \left(-\frac{z}{2} \right) G_{a\mu}^+ \left(\frac{z}{2} \right) \right| P - \frac{\Delta}{2} \right\rangle \Big|_{\substack{z^+=0 \\ z_\perp=0}}. \quad (2)$$

We note $\xi = -\Delta^+/(2P^+)$ the skewness variable and $t = \Delta^2$ the momentum transfer on the nucleon target. As usual, the "+" component refers to the projection of a 4-vector on a light-like vector n . The average momentum P obeys $P^2 = M^2 - t/4$, where M denotes the nucleon mass.

Similarly, in the polarized (axial-vector) sector, spin-1/2 GPDs are defined from:

$$\tilde{F}^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle P + \frac{\Delta}{2} \left| \bar{q} \left(-\frac{z}{2} \right) \gamma^+ \gamma_5 q \left(\frac{z}{2} \right) \right| P - \frac{\Delta}{2} \right\rangle \Big|_{\substack{z^+=0 \\ z_\perp=0}}, \quad (3)$$

$$\tilde{F}^g(x, \xi, t) = -\frac{i}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \left\langle P + \frac{\Delta}{2} \left| G_a^{+\mu} \left(-\frac{z}{2} \right) \tilde{G}_{a\mu}^+ \left(\frac{z}{2} \right) \right| P - \frac{\Delta}{2} \right\rangle \Big|_{\substack{z^+=0 \\ z_\perp=0}}. \quad (4)$$

Both F^a and \tilde{F}^a ($a = q, g$) can be decomposed as:

$$F^a = \frac{1}{2P^+} (h^+ H^a(x, \xi, t) + e^+ E^a(x, \xi, t)), \quad (5)$$

$$\tilde{F}^a = \frac{1}{2P^+} (\tilde{h}^+ H^a(x, \xi, t) + \tilde{e}^+ E^a(x, \xi, t)), \quad (6)$$

where the Dirac spinor bilinears are:

$$h^\mu = \bar{u} \left(P + \frac{\Delta}{2} \right) \gamma^\mu u \left(P - \frac{\Delta}{2} \right), \quad (7)$$

$$e^\mu = \frac{i\Delta_\nu}{2M_h} \bar{u} \left(P + \frac{\Delta}{2} \right) \sigma^{\mu\nu} u \left(P - \frac{\Delta}{2} \right), \quad (8)$$

$$\tilde{h}^\mu = \bar{u} \left(P + \frac{\Delta}{2} \right) \gamma^\mu \gamma_5 u \left(P - \frac{\Delta}{2} \right), \quad (9)$$

$$\tilde{e}^\mu = \frac{\Delta^\mu}{2M_h} \bar{u} \left(P + \frac{\Delta}{2} \right) \gamma_5 u \left(P - \frac{\Delta}{2} \right), \quad (10)$$

allowing the identification of the four GPDs H , E , \tilde{H} and \tilde{E} . The spinors are normalized so that $\bar{u}(p)\gamma^\mu u(p) = 2p^\mu$.

In principle, GPDs depend on a renormalization scale μ_R and a factorization scale μ_F , which are usually set equal to each other. From the point of view of code writing, we however keep two different variables, even we have took them equal in all applications so far.

1.2 Experimental access to Generalized Parton Distributions

GPDs are accessible in exclusive channels, where properties of all final state particles are reconstructed. Three of such channels attract most of the current experimental interest: Deeply Virtual Compton Scattering (DVCS), Timelike Compton Scattering (TCS) and Deeply Virtual Meson Production (DVMP).

DVCS was early recognized as the theoretically cleanest channel to access GPDs [1–3]. Pioneering studies [4, 5, 13] demonstrated the feasibility of such measurements at the beginning of the 21th century. Numerous dedicated experimental campaigns followed [14–33].

During the same period, an intense theoretical activity put DVCS under solid control. In particular we mention the full description of DVCS up to twist-3 [34–37], the computation of higher orders in the perturbative QCD expansion [38–47], the soft-collinear resummation of DVCS [48, 49], the discussion of QED gauge invariance [50–53] and the elucidation of finite- t and target mass corrections [54, 55].

1.2.1 Theory of Deeply Virtual Compton Scattering

We consider the photon lepto-production (l, l', γ) reaction on a target nucleon N :

$$l(k, h_l) + N(p, h) \rightarrow l(k', h_l') + N(p', h') + \gamma(q', \lambda'), \quad (11)$$

where the letters in parenthesis are the momenta and helicities of the particles. The amplitude \mathcal{T} for this process is the coherent superposition of the Deeply Virtual Compton Scattering (DVCS) and Bethe-Heitler (BH) amplitudes:

$$|\mathcal{T}|^2 = |\mathcal{T}_{\text{BH}} + \mathcal{T}_{\text{DVCS}}|^2 = |\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2 + \mathcal{I}, \quad (12)$$

or in terms of Feynman diagrams:

$$\sigma(ep \rightarrow ep\gamma) = \left| \underbrace{\text{DVCS}} + \underbrace{\text{Bethe-Heitler}} \right|^2. \quad (13)$$

We note $q = k - k'$ the momentum of the virtual photon of the VCS amplitude and:

$$Q^2 = -q^2, \quad (14)$$

$$x_B = \frac{Q^2}{2p \cdot q}, \quad (15)$$

$$t = (p - p')^2. \quad (16)$$

The corresponding cross-section is five-fold differential in x_B , Q^2 , t , and two azimuthal angles measured relatively to the lepton scattering plane, the angle ϕ to the photon-target scattering plane and the angle ϕ_S to the transversal component of the target polarization vector, as displayed on Fig. 1.

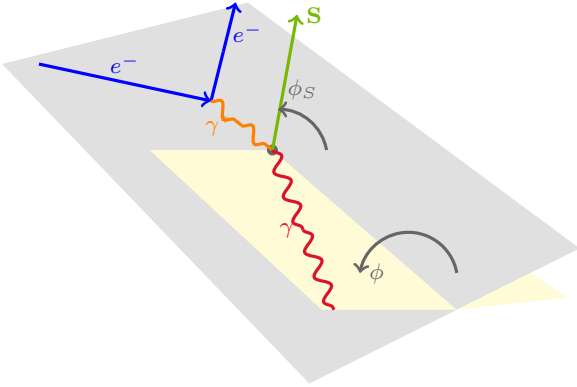


Fig. 1 Kinematics of DVCS: angular variables.

1.2.2 Factorization and coefficient functions

The Bjorken limit is defined by:

$$Q^2 \rightarrow \infty \text{ at fixed } x_B \text{ and } t. \quad (17)$$

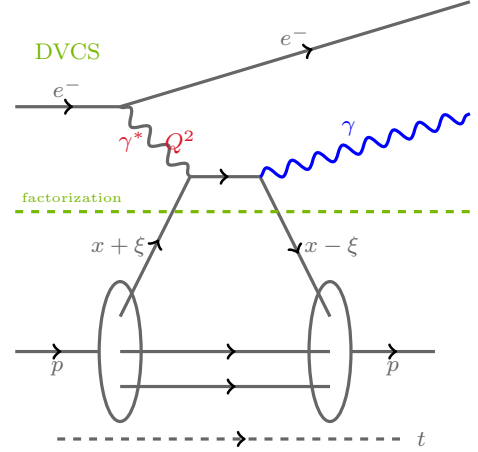


Fig. 2 Partonic interpretation of the DVCS process.

This corresponds to the factorization condition for the VCS amplitude [2, 41, 56, 57], which provides a *partonic* interpretation to the *hadronic* process: it is possible to reduce the reaction mechanism to the scattering of a virtual photon on one *active* quark (see Fig. 2).

The DVCS amplitude $\mathcal{T}_{\text{DVCS}}$ can be decomposed either in twelve helicity amplitudes or, equivalently, in twelve complex-valued Compton Form Factors (CFFs), which are usually denoted as \mathcal{H} , \mathcal{E} , $\tilde{\mathcal{H}}$, $\tilde{\mathcal{E}}$, \mathcal{H}_3 , \mathcal{E}_3 , $\tilde{\mathcal{H}}_3$, $\tilde{\mathcal{E}}_3$, \mathcal{H}_T , \mathcal{E}_T , $\tilde{\mathcal{H}}_T$, $\tilde{\mathcal{E}}_T$, with symbols reflecting their relation to GPDs. The last eight CFFs (\mathcal{F}_3) are related to the twist-three and transversity (\mathcal{F}_T) contributions, and disregarded in present analysis of DVCS data.

To keep the discussion simple, we will now focus on the GPD H and associated CFF \mathcal{H} . After proper renormalization, the CFF \mathcal{H} reads in its factorized form (at factorization scale μ_F):

$$\mathcal{H} = \int_{-1}^1 dx \left[\sum_q^{n_F} T^q(x) F^q(x) + T^g(x) F^g(x) \right], \quad (18)$$

where the explicit skewness dependence was omitted. The renormalized coefficient functions are given by:

$$T^q(x) = \left[C_0^q(x) + C_1^q(x) + \ln \left(\frac{Q^2}{\mu_F^2} \right) C_{\text{coll}}^q(x) \right] - (x \rightarrow -x), \quad (19)$$

$$T^g(x) = \left[C_1^g(x) + \ln \left(\frac{Q^2}{\mu_F^2} \right) C_{\text{coll}}^g(x) \right] + (x \rightarrow -x) \quad (20)$$

We will only show results of the Next to Leading Order (NLO) calculations of the quark coefficient functions to exhibit their structure, and refer to the literature for the gluon coefficient functions [38, 40–46]. The

quark coefficient functions write:

$$C_0^q(x, \xi) = -e_q^2 \frac{1}{x + \xi - i\epsilon}, \quad (21)$$

$$C_1^q(x, \xi) = \frac{\alpha_s C_F}{4\pi} C_0^q(x, \xi) \left[3 \frac{x + \xi}{x - \xi} \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right) + \ln^2 \left(\frac{x + \xi}{2\xi} - i\epsilon \right) - 9 \right], \quad (22)$$

$$C_{\text{coll}}^q(x, \xi) = \frac{\alpha_s C_F}{4\pi} C_0^q(x, \xi) \left[3 + 2 \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right) \right], \quad (23)$$

where $C_F = (N_c^2 - 1)/(2N_c)$, N_c being the number of colors, and e_q is the quark electric charge in units of the positron charge. The coefficient C_0^q is the Leading Order (LO) contribution.

1.2.3 Observables of the DVCS channel

The cross section of electroproduction of a real photon off an unpolarized target can be written as:

$$d\sigma^{h_l, e_l}(\phi) = d\sigma_{\text{UU}}(\phi) [1 + h_l A_{\text{LU, DVCS}}(\phi) + e_l h_l A_{\text{LU, I}}(\phi) + e_l A_{\text{C}}(\phi)], \quad (24)$$

where e_l is the beam charge (in units of the positron charge) and $h_l/2$ the beam helicity. If longitudinally polarized, positively and negatively charged beams are available, the asymmetries in Eq. (24) can be isolated. This is the case for a large part of the data collected by HERMES. For example, the beam charge asymmetry is obtained from the combination:

$$A_{\text{C}}(\phi) = \frac{1}{4d\sigma_{\text{UU}}(\phi)} \left[(d\sigma^{\rightarrow\rightarrow} + d\sigma^{\rightarrow\leftarrow}) - (d\sigma^{\rightarrow\rightarrow} + d\sigma^{\rightarrow\leftarrow}) \right]. \quad (25)$$

From similar combinations, we obtain the two beam spin asymmetries $A_{\text{LU, I}}$ and $A_{\text{LU, DVCS}}$:

$$A_{\text{LU, I}}(\phi) = \frac{1}{4d\sigma_{\text{UU}}(\phi)} \left[(d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}) - (d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}) \right], \quad (26)$$

$$A_{\text{LU, DVCS}}(\phi) = \frac{1}{4d\sigma_{\text{UU}}(\phi)} \left[(d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}) + (d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}) \right]. \quad (27)$$

However, if an experiment cannot change the value of the electric charge of the beam (such as in Jefferson Lab), the asymmetries defined in Eq. (24) cannot be isolated any more, and one can only measure the following (total) beam spin asymmetry $A_{\text{LU}}^{e_l}$:

$$A_{\text{LU}}^{e_l}(\phi) = \frac{d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}}{d\sigma^{\rightarrow\rightarrow} + d\sigma^{\rightarrow\leftarrow}}, \quad (28)$$

where we classically denote by " \pm " the sign of the beam charge e_l , and by an arrow \rightarrow (\leftarrow) the helicity plus (minus). This definition of $A_{\text{LU}}^{e_l}$ can be expressed as a function of the spin and charge asymmetries defined in Eq. (24):

$$A_{\text{LU}}^{e_l}(\phi) = \frac{e_l A_{\text{LU, I}}(\phi) + A_{\text{LU, DVCS}}(\phi)}{1 + e_l A_{\text{C}}(\phi)}. \quad (29)$$

We refer to Ref. [58] for a systematic nomenclature of DVCS observables and their relations to CFFs. Different observables being related to different combinations of CFFs with different weighting factors, a flexible GPD code should not only be able to deal with different exclusive channels, but also with cross sections and various asymmetries.

2 Needs assessment

2.1 From GPDs to observable: basic structure

The basic structure of the computation of an observable of one channel related to GPDs is outlined in Fig. 3. We illustrate the situation in the DVCS case, but the following considerations should apply to any channel. The *nonperturbative* level contains GPDs as functions of x , ξ , t , μ_F and μ_R , but also as functions of unspecified (model-dependent) parameters. The dependence on the factorization scale μ_F is described by evolution equations. The kernels of the GPD evolution equations at LO were derived in the seminal papers introducing GPDs or soon after [1, 3, 56, 59, 60]. The kernels at NLO were obtained in Refs. [61–65]. The corresponding work for transversity GPDs was published in Refs. [66–68]. To stay as generic as possible, evolution equations should be solved mostly in x -space, but with different numerical integration routines if we require speed and/or accuracy. The *perturbative* level convolutes GPDs with various coefficient functions depending on the considered channel (see *e.g.* Sec. 1.2.2). Here, again we should be free to select the integration routine fulfilling our needs. Various theoretical frameworks exist that take into account *e.g.* the target mass and finite- t corrections [54, 55], the soft-collinear resummation of DVCS [48, 49], higher order effects either in the coefficient function [38–42] or in the evolution kernel [61–65]. All theoretical frameworks should work all the same with a given GPD model. The *full process* level produces cross sections or asymmetries (see *e.g.* Sec. 1.2.3) for various kinematics. For fitting purposes, all observables (whatever the channel is) should be treated in the same manner in order to simplify handling of experimental data. We may want to check *e.g.* the impact of one specific data set on the general knowledge of GPDs,

or to apply some kinematic cuts in order to guarantee that the analysis takes place in a range where factorization theorems apply. Note that if we want to fit the data (say, if we want to minimize a χ^2), then we will have to loop over this GPD-to-observable structure at each step of the minimization.

2.2 Needs and constraints

The basic structure of the computations, the type of studies to be done, or simply the profile of the users, already put strong constraints on the software architecture design.

First of all, maintaining the software platform, or adding new theoretical developments (*e.g.* the aforementioned recent computation of target mass and finite- t corrections) should be as easy as possible. As a consequence, the structure should be flexible enough to allow the manipulation of an important number of physical concepts of different nature. We may want to use the same tools to test new theoretical ideas, and to design new experiments. In that case, we want to be able to perform some differential studies in a robust and simple way to see what observable is most sensitive to the physical hypothesis in the modeling. Implicitly this means that the user of the code will probably know only remotely the detailed description of the physical module he is using: we cannot expect any user to be an expert in any physical models involved in the platform. However, a careful user should get a correct result, even without knowing all details of the implementation. This means that all that *can be* automated *has to be* automated, and that physical modules should be designed in such a way that an inadequate use is forbidden, or indicated to the user with an explicit warning.

Second, with respect to maintenance, we want to be sure that adding new functionalities or new modules will not do any harm to the existing pieces of code. This requires some non-regression tools to guarantee that the version $n + 1$ has (at least) all the functionalities of version n . To trace back the results of the code (*e.g.* to be able to reproduce the results of a fit), it should be possible to save some *computing scenarios* for later reference. The maintenance of the PARTONS code on a long-term perspective is one of the key element of its design, aimed at robustness and flexibility. It was developed following *agile development procedures* structured in cycles, with intermediate deliverables and a functioning architecture all way long.

Third, the code should ideally be used by a heterogeneous population of users, ranging from theoretical physicists used to symbolic computation softwares, to

experimentalists using the CERN library ROOT [69] and Monte Carlo techniques to design new experiments.

Ideally, it should have been possible to run the code through a web interface, in the spirit of the **Durham service** for PDFs. However, such a solution requires dedicated work to synchronize a database to a web page, to prevent the web page of any attack, to create a queue system if several users want to perform a computation at the same time and to handle large volumes of data, which can always happen with functions depending on (at least) the three **double** variables x , ξ and t . In particular, this means that a dedicated engineer has to take part of his time to tackle these problems and maintain all basic functionalities. In a general context of budget cuts, it is not clear that such a solution is realistic on a long-term perspective. We therefore choose to let users download a client application to run the code on their own machines.

3 Code Development

The PARTONS platform is written in C++. This choice has been made for performances and to have an homogeneous product in terms of coding and programming languages. In particular, there is no wrapping of other third-party software written in a different programming language. The project considers two different communities: the developers, who have to understand the software architecture to use low-level functions, and the users, who can just use high-level functions ignoring the details of implementations. With the progress of automation, the users may run the code without writing any line of C++. In the community of developers, a crucial role is played by the software architect, who is responsible for the integration of new modules in the platform. He guarantees the robustness and homogeneity of the code being developed. We decided to depend as little as possible on third-party libraries to help its dissemination. So far only the dependence on the cross-platform application framework **Qt** is essential because it manages the connections to different types of databases in a generic way (see Sec. 4.5). However the PARTONS code still contains residual dependencies that may easily be suppressed later. These are: the **Class Library for Numbers (CLN)**, the CERN library **ROOT** and the **GNU Scientific Library (GSL)**.

3.1 Development tools

In order to allow concurrent development, the code has been synchronized to a **subversion** system that is hosted by CEA Saclay. The system allows to access the

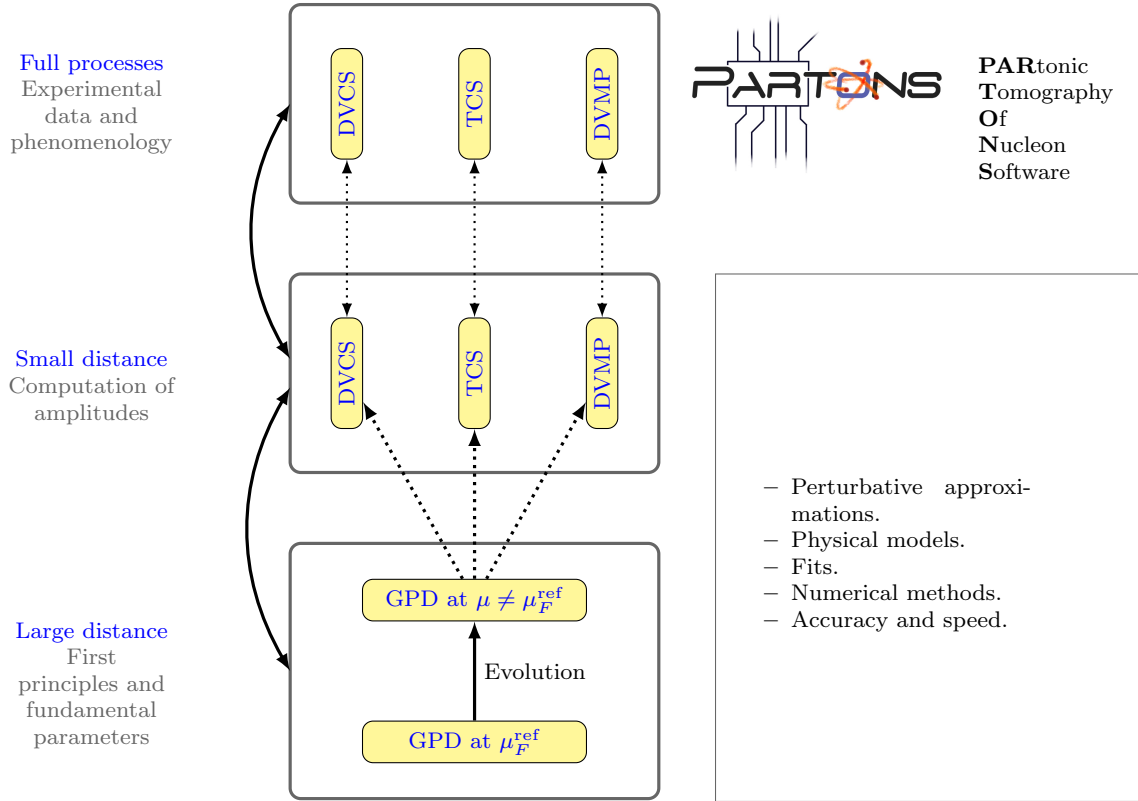


Fig. 3 The computation of an observable in terms of GPDs is generically layered in three basic steps: description of the hadron structure with nonperturbative quantities, computation of coefficient functions, and evaluation of cross sections.

code by any authorized developer, also outside Saclay, and it stores the history of changes. As a result, one missing developer does not slow down the whole team by creating a lack of information. The code is managed with the web-based management system **trac** that offers, among others, a wiki for knowledge management and transfer, as well as a bug report system. Coding conventions, variables naming schemes, commentaries, *etc.* has been homogenized with an Integrated Development Environment (IDE): **Eclipse**.

To obtain the first complete DVCS computing chain in a reasonable amount of time, we decided to rationalize the development environment by providing all developers with a virtual machine running the same Linux distribution, with the same configuration of Eclipse, and the same version of the third party libraries. When releasing the code to a vast community of users, we should provide multi-platform solutions. But to obtain a proof of capabilities, a working DVCS computing chain running on a single specific environment does not spoil the demonstration.

3.2 Logger

The PARTONS code is composed of two threads: one in charge of the execution of the computations, and the other one serving as a *logger*. The code sends information at four different levels: DEBUG, INFO, WARNING and ERROR. With the first three levels the code is always running, while the ERROR level forces the code to stop. Sending the information to the logger does not slow down the computations by taking precious time to screen (or file) printing. Warning messages signal to the user that there is something to be checked carefully, *e.g.* slow convergence, *etc.* The output of the logger can be directed either to the screen, or to a log file, or both. It traces back all details of the computation that have been considered relevant by the developer.

3.3 Non-regression and testing

A companion project, PARTONS-Tests, contains all tests related to the objects developed in PARTONS. When a new module is created, the developers devise

some tests it has to pass. Before releasing a new version of the code we run all such tests, which either say that a test is passed without giving any details, or say that a test is failed, and provide information about it. This is extremely important to ensure the code *non-regression*. We were also careful to keep track of the testing perimeter, for example, the range of GPD variables (x, ξ, t) used in the validations.

4 Layered and service-oriented architecture

From the software engineering point of view, the PARTONS project benefits from a layered and service-oriented architecture, which provides flexibility and standardization. To the best of our knowledge, this architecture is original in the world of scientific computing, at least in nuclear and particle physics. It is derived from web-oriented technologies such as the **Java EE** specification. We describe below the whys and hows of these choices.

4.1 Layers

Ideally the code should not have to go through a major rewriting during the years dedicated to the analysis of Jefferson Lab and COMPASS GPD data. One way to ensure this is to isolate potential modifications as well as possible. This is the reason for the layered architecture: every part of the architecture belongs to a *layer*, and a modification in a layer does not hinder other layers.

The layered structure of the PARTONS software is shown in Fig. 4, and is made of seven parts. Layer 3 contains all modules. This layer contains the physics engine: GPD models, but also computations of coefficient functions, or DVCS cross sections with different approximations on the treatment of kinematic terms. A module is fed by data, and outputs results. Layers 2 and 5 correspond to data and results respectively. In these layers no treatment is made on data. There are just containers (high-level objects) that make sure, for example, that each module receives an object which has been well-formed thanks to its constructor. Instead of feeding a GPD module with 5 **double** variables $(x, \xi, t, \mu_F$ and $\mu_R)$ which can be sent in an incorrect order after some minor editing in the code, we isolate the places where that kind of error may happen instead of spreading them in all calls to a GPD module throughout the code. We trust the fact that the high-level object has been correctly constructed from these five **double** variables. The risk of an accidental manipulation (*e.g.* the exchange of μ_F and t) becomes much more lim-

ited. To illustrate this, we provide the code defining the **GPDKinematic** class:

```

1 class GPDKinematic: public Kinematic {
2 public:
3     /**
4      * Default constructor
5      */
6     GPDKinematic();
7
8     /**
9      * Constructor
10     */
11     * @param x Longitudinal momentum fraction of the active
12     parton
13     * @param xi Skewness
14     * @param t Mandelstam variable, momentum transfer on the
15     hadron target (in GeV^2)
16     * @param MuF2 Factorization scale (in GeV^2)
17     * @param MuR2 Renormalization scale (in GeV^2)
18     */
19     GPDKinematic(double x, double xi, double t, double MuF2,
20     double MuR2);
21
22     GPDKinematic(ParameterList &parameterList);
23
24     /**
25     * Return a pre-formatted characters string for output
26     visualization of class member's values
27     */
28     * @return a pre-formatted characters string
29     */
30     virtual std::string toString() const;
31
32 private:
33     KinematicType::Type m_kinematicType;
34
35     double m_x;          ///< Longitudinal momentum fraction of
36     the active parton
37     double m_xi;         ///< Skewness
38     double m_t;          ///< Mandelstam variable, momentum
39     transfer on the hadron target (in GeV^2)
40     double m_MuF2;       ///< Factorization scale (in GeV^2)
41     double m_MuR2;       ///< Renormalization scale (in GeV^2)

```

It contains the variables x, ξ, t, μ_F^2 and μ_R^2 , and three methods to:

1. create the object from a set of five **double** variables,
2. create it from a generic list of parameters **ParameterList** (for automation),
3. return a **string** to print the content of the object on screen or in a file.

We emphasize that this structure is not specific to GPD modules. Every family of modules has its own input and output types, which are generically referred to as *beans*. Storing all input variables in complex objects also makes sure that, for example, a GPD will not accidentally be evaluated at something completely different, such as an angular variable (still a **double** variable) to describe a DVCS kinematic configuration.

Another critical layer (layer 4) contains services. Services basically link related modules to offer high-level functions to the user, and help hiding the complexity of low-level functions. The whole code can however be used without services, but it is less convenient.

At last, three extra layers provide useful functionalities to the users: layer 6 contains the tools to store the results in a database. It is made so as to optimize later

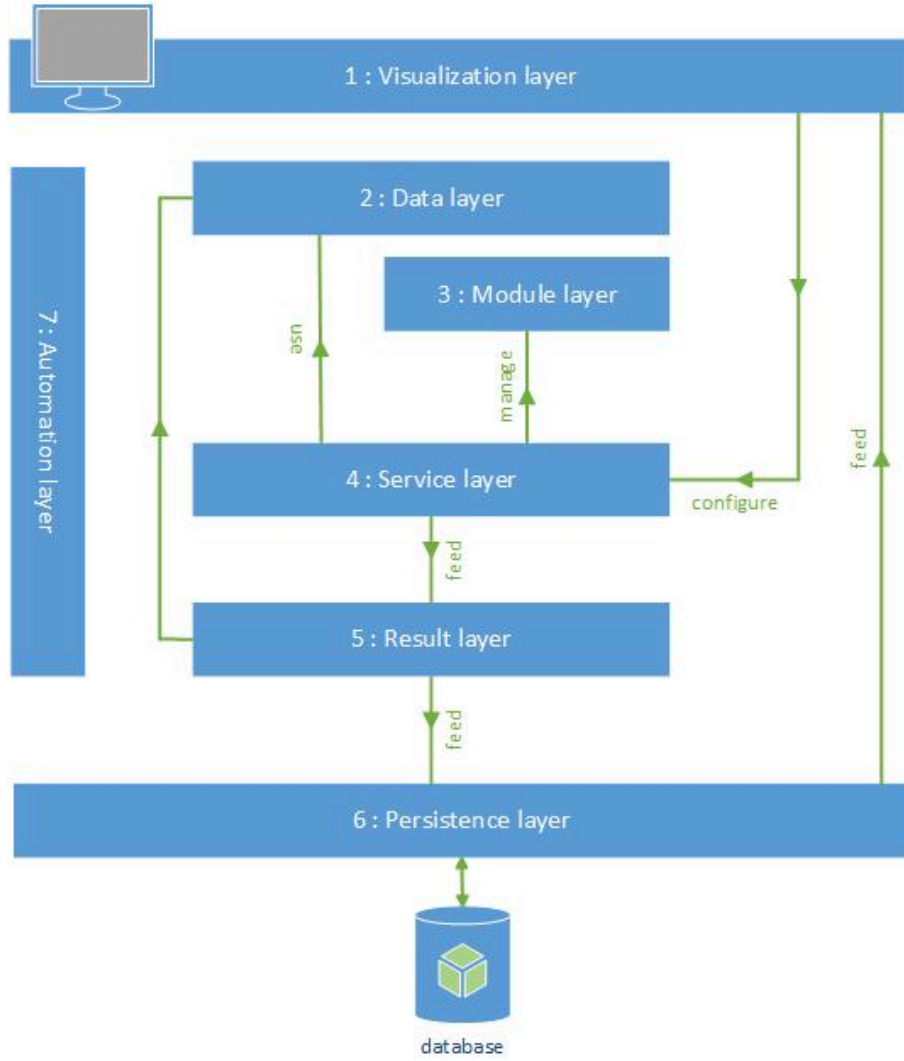


Fig. 4 The basic layer structure of the PARTONS project. The numbering scheme of the layers reflects the call sequence of the different parts of the architecture. Eventually the user will be able to launch computations from the client visualizing interface.

requests and post-processing treatments, and to limit the data redundancies in the database. Layer 1 integrates all visualizing tools: through an interface, the user makes some requests to the database containing the output of the code, and draw curves on screen and produce grids of points in files. The last layer (layer 7) contains all automation tools. A scenario, *i.e.* a file containing all physical or mathematical assumptions on the computation to be performed, is parsed. All relevant objects are created, and evaluations are processed. The results can then stored in the database with the associated scenario (if the user chooses so), either to trace back all hypothesis underlying the results, or to be able to evaluate them again later, *e.g.* for non-regression purposes.

4.2 Modules

The flexibility of the architecture is achieved through class inheritance. The logical sequence of the code as a whole is centralized in classes which receive *standard* inputs and return *standard* outputs. The details of model description, or numerical resolution, *etc.*, are left to daughter classes.

An example is provided by Fig. 5 which describes the actual implementation for GPD modules. The input is a `GPDKinematic` object described above. The output is an object called `GPDResult` which contains, for all GPDs F provided by the considered model, the gluon GPD F^g and the singlet and nonsinglet combinations of quark GPDs $F^q(x)$, $F^{q(+)}(x) = F^q(x) - F^q(-x)$ and $F^{q(-)}(x) = F^q(x) + F^q(-x)$ if $F = H$ or E . This out-

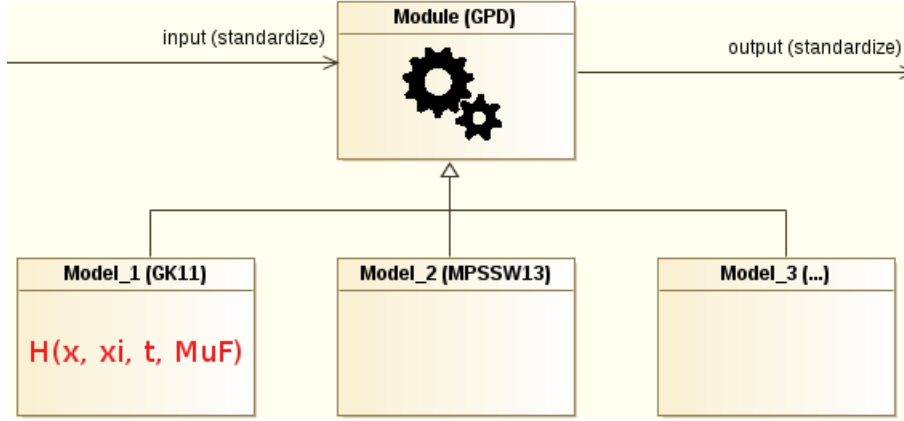


Fig. 5 Modularity through class inheritance and standardized inputs and outputs

put object also contains functions to filter the data (*e.g.* depending on the parton type) or to print the results. At last, a `GPDModule` generically depend on "kinematic" variables x , ξ , t , μ_F and μ_R , and on a list of parameters, depending on the specific implementation of the model. A `GPDModule` is a collection of methods to compute various GPDs (*e.g.* H , E , *etc.*). There is no upper or lower limits on the number of GPD types a `GPDModule` should contain.

```

1  /**
2   * Virtual method, computes GPD with some input parameters.
3   *
4   * @param x Longitudinal momentum fraction
5   * @param xi Skewness
6   * @param t Momentum transfer (Mandelstam variable)
7   * @param MuF2 Factorisation scale (in GeV^2)
8   * @param MuR2 Renormalisation scale (in GeV^2)
9   * @param gpdComputeType H, Ht, E, Et, ... or ALL. See
10  GPDComputeType for
11  more details.
12  * @return Return results in an GPDOutputData class.
13  * Contains GPD results for each flavor of quarks and for each
14  GPDs (H, Ht,
15  E, Et, ...) if computable.
16  */
17  virtual GPDResult compute(double x, double xi, double t,
18  double MuF2,
19  double MuR2, GPDType::Type gpdType);
20  virtual PartonDistribution computeH();
21  virtual PartonDistribution computeE();
22  virtual PartonDistribution computeHt();
23  virtual PartonDistribution computeEt();

```

The variable `gpdType` selects the type of GPD to be computed, *i.e.* H , E , \dots , or all GPDs available in the considered model. Extending a model by adding GPDs is then fairly simple. It suffices to inherit from a class, and to add the appropriate "compute", *e.g.* `computeHT` for the transversity GPD H_T . Daughter classes contain specific implementations, corresponding *e.g.* to the GK [70–72] or VGG [7, 73–75] models. Any model obeying this general structure can enter the PARTONS platform and benefit from all other functionalities.

The example shown above for a GPD model can be extended to any other module, such as QCD evolution modules, DVCS observable modules, *etc.*

4.2.1 Registry

Adding a new daughter class does not require any modifications of the existing code as long as this class inherits from an existing module. Concretely, we can freely add as many GPD models as we want. On the contrary, if we wish to extend the functionalities of the PARTONS platform to the computation of Transverse Momentum Dependent parton distribution functions (TMDs), we will have to create all parent classes to define what a TMD is. Adding a new module simply consists in adding a new file to the whole project. The interoperability of the PARTONS structure is thus maintained all way long. This essential feature is provided by the Registry.

The *Registry* is the analog of a phone book, which lists all available modules. From the point of view of software engineering, it corresponds to the *singleton* design pattern which ensures that it is unique. When a new module is created, the first thing to do is to call this unique instance, and to register the new module with the class name provided by the developer of the module (see Fig. 6). In return the Registry gives a *unique* identifier encoded in a `int` variable for performance purposes.

The following code corresponds to the implementation of the 2011 version of the GK model:

```

1 #include " ../BaseObjectRegistry.h"
2
3 // Initialize [class]::classId with a unique name
4 const unsigned int GK11Model::classId = BaseObjectRegistry::
  getInstance()->registerBaseObject(new GK11Model("GK11Model1"));

```

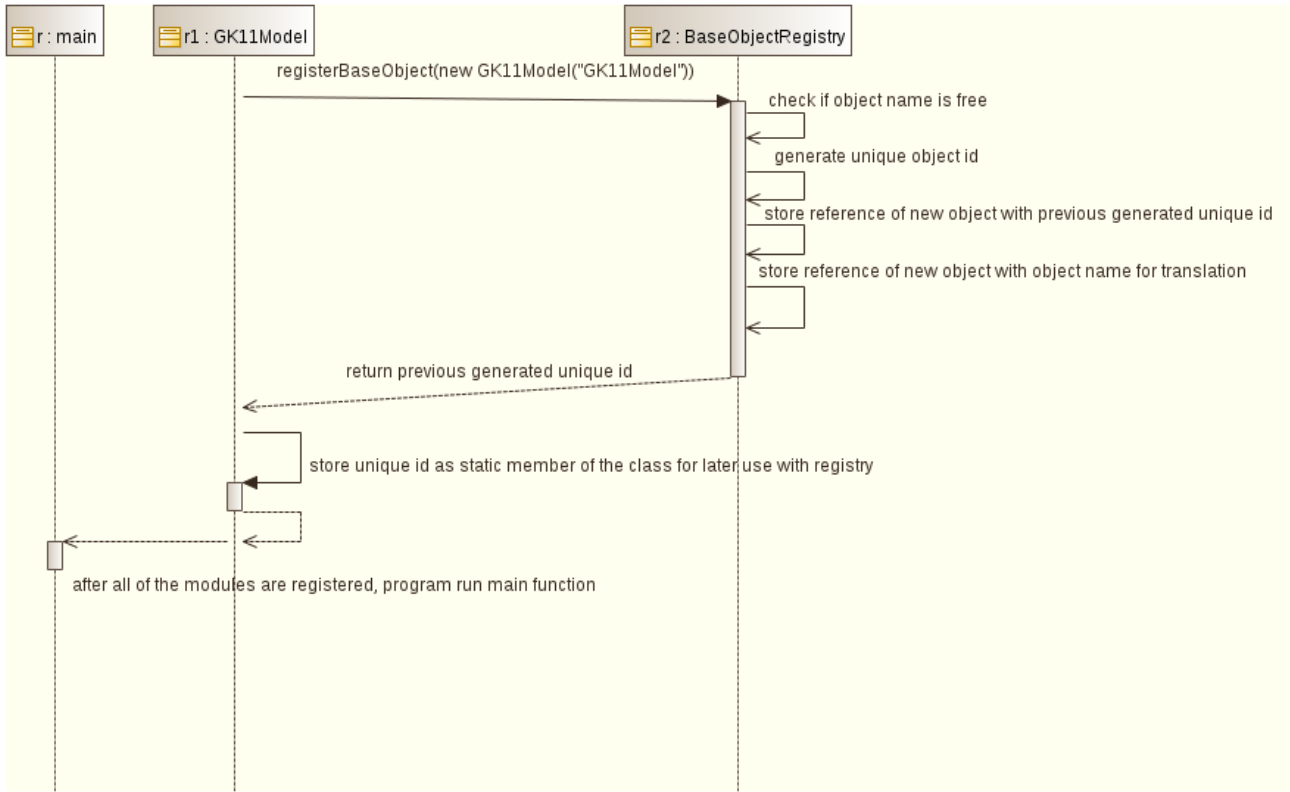


Fig. 6 Sequence diagram presenting the different steps, arranged in time, allowing the self-registration of all modules at the start of the execution of the PARTONS code. Parallel vertical lines (lifelines) represent the different processes or objects simultaneously living. Vertical boxes indicate the duration of the process between the function call and return. Full lines with arrows signal function calls and dotted lines with arrows function returns. Staples show the life cycle of specific objects.

This version of the GK model can then be used everywhere with its identifier: `GK11Model::classId`. This identifier is a static variable, *i.e.* it is the same throughout the whole platform and for all instances of the `GK11Model` class. Being a static variable, the identifier is resolved prior to the execution of the `main` of the code. The identifier being unique and registered prevent undesirable code operating. For example, if a user accidentally asks for the GK12 model instead of GK11, the code will simply not compile. This would not have been achievable if modules were identified by simple types as `strings`.

At this stage, it is important to mention that the Registry stores pointers to all modules in a generic way, *i.e.* whatever their nature: pointers to `GPDModule`, to `RunningAlphaStrongModule`, to `DoubleDistributionModule`, *etc.* This is achieved by requiring all modules to derive from a single parent class named `BaseObject`. `BaseObject` is the "zeroth-level-object" of the architecture. Any C++ object in PARTONS can inherit of it. It also carries information on the identity of a specific object to transmit an explicit message to the logger (*i.e.*

understandable by a human being, not an address in memory).

4.2.2 Factory

The Registry lists everything that is available in the platform, but only one species of each. If one wants to use a module, one cannot take it from the Registry, otherwise it would not be available anymore. The solution consists in using the *Factory* design pattern, which gives to the user a pre-configured copy of an object stored in the Registry. The user can then manage the configuration of the module and its life cycle.

The principle of a Factory is the following. We consider once again the `GK11Model`. By construction it is derived from `GPDModule`, which itself is derived from `BaseObject` to be stored generically in the Registry. As show on Fig. 7, when the user wants to use the GPD model with identifier `GK11Model::classId`, he asks the `ModuleObjectFactory` to return him a pointer to a `GPDModule`. The `ModuleObjectFactory` asks the `BaseObjectFactory` to provide a new instance of a `GPDModule` with type `GK11Model`. To this aim, `BaseObject`

Factory requests that the Registry gives back the reference to the `GK11Model` already stored in memory. The Registry goes through its internal list to find the `BaseObject` with identifier `GK11Model::classId`. Using the returned reference, the `BaseObjectFactory` clones¹ the `GK11Model` object and provides the `ModuleObjectFactory` a reference to the duplicated object. Finally the `ModuleObjectFactory` runs a `static_cast` to give back a pointer to the appropriate type `GPDMModule` instead of a `BaseObject` type. What is needed to fit to the structure of the code is a `GPDMModule` (seen from the exterior of a black box, the GPD models are all the same object), but the specific implementation (*i.e.* what *defines* a model from the physics point of view, and what is in the black box from the software point of view) is in a daughter class, like `GK11Model`, *etc.*: through pointers and inheritance, the *polymorphism* feature of C++ allows the selection of a given module at runtime.

This is the basic sequence underlying the automation of the PARTONS code, discussed below in Sec. 4.4. This works *mutatis mutandis* for all modules.

4.3 Services

Services hide the complexity of low-level functions to provide high-level functionalities to the user. A service is basically a toolbox for the user: the user is being given tools to use the software without knowing the details of its operating². Services demonstrate their relevance in computations that combine several different objects. Before the refactoring of our GPD codes to the PARTONS platform, the user had to take the outputs from various objects, like *e.g.* some GPD values, to manually run an evolution code and then feed the code computing CFFs. These operations were hand-made, with all the risks this implies. In the PARTONS structure, services combine different modules and data sets to produce results in a transparent way. Among others, the `GPDServices` provide several functions that hide the complexity related to repetitive tasks, like evaluating a GPD for a list of kinematic configurations, or evaluating several GPD models at the same input kinematic. The following excerpt shows what are the presently offered options:

```
1 class GPDServices: public ServiceObject {
2 public:
3     static const unsigned int classId; ///< Unique ID to
    automatically register the class in the registry.
```

¹It is not a copy of a pointer, which would still points to the same object. It is a *duplication* of the object, referred to by a new pointer.

²As an image, we can say that we can start a car by turning a key, and not knowing the detailed description of the motor, and of electric circuits between the motor and the key.

```
4     virtual void computeTask(Task &task);
5
6
7     /**
8      * Computes GPD model at specific kinematic
9      *
10     * @param gpdKinematic
11     * @param pGPDMModule
12     * @return
13     */
14     GPDResult computeGPDModel(const GPDKinematic &
    gpdKinematic,
15                               GPDMModule* pGPDMModule);
16
17     /**
18     * Computes the GPD model at specific kinematic, restricted by
19     a GPD's type
20     *
21     * @param gpdKinematic
22     * @param pGPDMModule
23     * @param gpdType
24     * @return
25     */
26     GPDResult computeGPDModelRestrictedByGPDDType(
    const GPDKinematic &gpdKinematic, GPDMModule*
27     pGPDMModule,
28     GPDDType::Type gpdType);
29
30     /**
31     * Computes GPD model at specific kinematic with a QCD
32     evolution model.
33     *
34     * @param pGPDMModule
35     * @param pEvolQCDModule
36     * @param gpdKinematic
37     * @param gpdType
38     * @return
39     */
40     GPDResult computeGPDModelWithEvolution(const
    GPDKinematic &gpdKinematic,
41     GPDMModule* pGPDMModule, GPEvolutionModule*
42     pEvolQCDModule,
43     GPDDType::Type gpdType = GPDDType::ALL);
44
45     /**
46     * Computes GPD model by a kinematic list
47     *
48     * @param listOfGPDKinematic
49     * @param pGPDMModule
50     * @return
51     */
52     GPDResultList computeListOfKinematics(
    std::vector<GPDKinematic> &listOfGPDKinematic,
    GPDMModule* pGPDMModule);
```

Five different types of computations are currently available:

1. The function `computeTask` is generic and is one of the building blocks of automation in PARTONS.
2. `computeGPDModel` evaluates all GPDs in a given model at the same kinematics without using QCD evolution equations. The μ_F dependence is that which is built-in in the model, *e.g.* the factorization scale of the PDFs in Radyushkin Double Distribution Ansatz models [76].
3. `computeGPDModelRestrictedByGPDDType` evaluates a specific GPD (say H) at a given kinematics. It is a restricted version of the previous function.
4. `computeGPDModelWithEvolution` evaluates a specific GPD (say H) at a given kinematics using QCD evolution equations. By default all GPDs offered in the considered GPD model are evaluated.

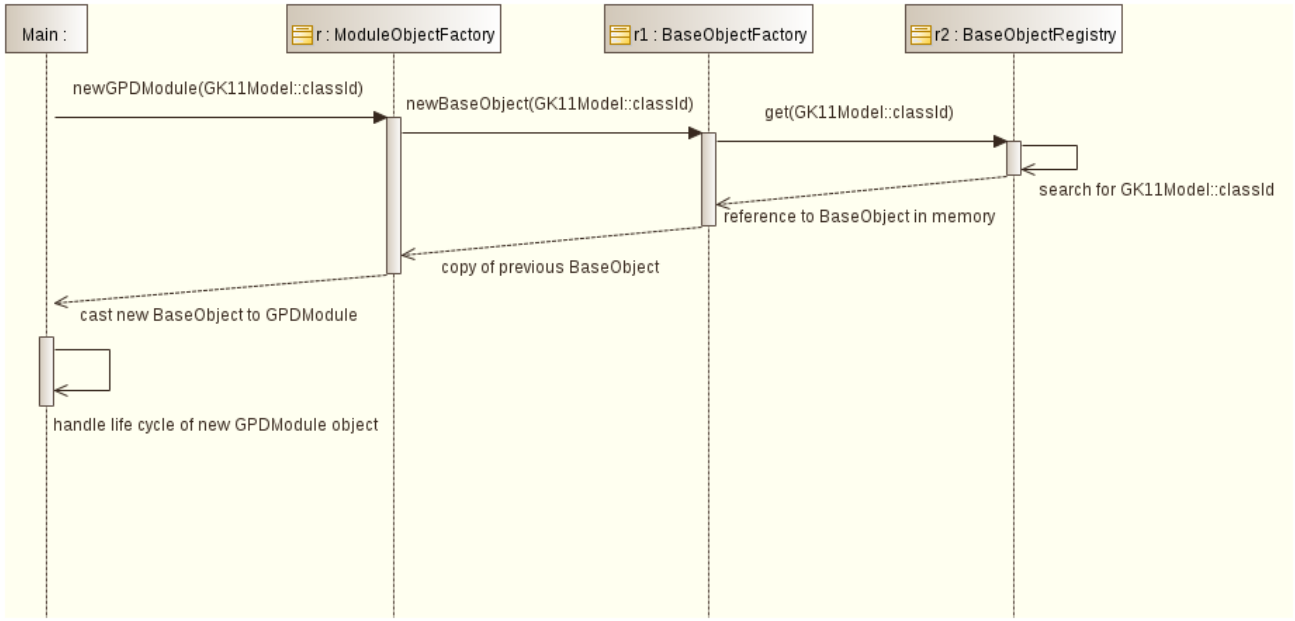


Fig. 7 Sequence diagram presenting the different processes, arranged in time, relating the different objects allowing the instantiation of a new module from the sole information of its identifier or name. Parallel vertical lines (lifelines) represent the different processes or objects simultaneously living. Vertical boxes indicate the duration of the process between the function call and return. Full lines with arrows signal function calls and dotted lines with arrows function returns. Staples show the life cycle of specific objects.

5. `computeListOfKinematics` evaluates all GPDs in a given model at a list of kinematic configurations. It is useful for example for systematic comparisons between theoretical predictions and experimental data, where measurements can be divided into tenths to hundredths of (x_B, t, Q^2) bins.

Thanks to the services, repetitive tasks are coded and validated once, which saves many possible implementation errors. For example, the code implementing the function `computeGPDModelRestrictedByGPDType`, is below:

```

1  GPDResult GPDServic::
   computeGPDModelRestrictedByGPDType(
2     const GPDKinematic &gpdKinematic, GPDModule*
   pGPDModule,
3     GPDType::Type gpdType) {
4
5     GPDResult gpdResult = pGPDModule->compute(
   gpdKinematic.getX(),
6     gpdKinematic.getXi(), gpdKinematic.getT(),
   gpdKinematic.getMuF2(),
7     gpdKinematic.getMuR2(), gpdType);
8
9     return gpdResult;
10 }

```

The service unfolds the `GPDKinematic` object to obtain the familiar variables x, ξ, t, μ_F and μ_R . As mentioned above, the role of the variable `gpdType` is the selection of the GPD to be computed through the function `compute` which is required in all objects inheriting from `GPDModule`. To trace back the results of an evaluation,

the object `GPDResult` contains not only the values of the GPDs, but also the kinematic points x, ξ, t, μ_F and μ_R , and the identifier of the GPD model that was used.

4.4 Automation and scenario manager

What the end-of-line user really wants is just selecting the various models, kinematic configurations and observables to compute by specifying the necessary physical hypothesis. At the end of the day, this should be accomplished with a simple file, or through a web page. This would allow the PARTONS software to be used by physicists unfamiliar to C++, which represents a significant part of the theoretical physics community. We decided to give to the code the appropriate information, a *scenario*, through a XML file. This offers several advantages. First the file can be read or manually written by simple adaptation. Being a markup language, its formal likeness to \LaTeX should make it look familiar to physicists. Second, it can be easily generated by a web interface. Third, the freedom in defining markups allows a structure very similar to that of the underlying C++ objects. For example the computation of a beam-spin asymmetry A_{LU} (see Sec. 1.2.3) reads:

```

1  <?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
2

```

```

3 <scenario id="04" date=""
4   description="Example of computation of one observable (Alu)
   with GV2008_DVCS_model">
5
6   <!-- storeInDB = "1" for storage of observable values in
   database, "0" otherwise -->
7   <task service="ObservableService" method="
   computeDVCSObservable" storeInDB="1">
8     <!-- Select MuF2 = MuR2 = lambda Q2 -->
9     <ScaleModule>
10      <param name="id" value="Q2Multiplier" />
11      <param name="lambda" value="1" />
12    </ScaleModule>
13
14    <!-- Define the relation between xB and xi -->
15    <XiConverterModule>
16      <param name="id" value="XBToXi" />
17    </XiConverterModule>
18
19    <!-- Specify kinematics -->
20    <ObservableKinematic>
21      <param name="xB" value="0.1763" />
22      <param name="t" value="-0.1346" />
23      <param name="Q2" value="1.3651" />
24      <param name="phi" value="22.5" />
25    </ObservableKinematic>
26
27    <!-- Specify observable -->
28    <Observable>
29      <param name="id" value="Alu" />
30    </Observable>
31
32    <!-- Specify set of DVCS cross section expressions and
   beam energy -->
33    <DVCSModule>
34      <param name="id" value="GV2008Model" />
35      <param name="beam_energy" value="5.77" />
36    </DVCSModule>
37
38    <!-- Specify computing methods of CFFs and pQCD
   accuracy -->
39    <DVCSConvCoeffFunctionModule>
40      <param name="id" value="DVCSFFModel" />
41      <param name="qcd_order_type" value="LO" />
42    </DVCSConvCoeffFunctionModule>
43
44    <!-- Specify GPD model -->
45    <GPDModule>
46      <param name="id" value="GK11Model" />
47    </GPDModule>
48  </task>
49 </scenario>

```

This XML file is parsed by an object named **Scenario Manager**, which now possesses a collection of **string** objects. The real difficulty is the creation of C++ objects from this collection of **string** objects. All scenarios start with the specification of a service and of a method inside that service. All **string** objects enclosed between the two task markups **<task service=...** and **</task>** are sent to the selected method in the service (**computeDVCSObservable** here). The **ObservableService** "knows" what the method really needs to perform the computation, and look through the lists of objects and parameters if all the relevant information is provided. The **ObservableService** possesses a **computeTask** method, and only services do have such methods. The role of the **computeTask** functions is the distribution of the **ParameterList** objects to the different constructors of the various objects required to perform the considered task. Centralizing the creation of all the

objects in the **computeTask** function of a service gives robustness to the generic use of an XML scenario. Then, for beans (inputs, outputs), everything is taken care of by the constructors. For modules, the code calls the Factory which will get the required objects from their name: precisely, the Factory gets the name, check in the Registry³ if there exists a pointer corresponding to that name, and either gives a copy of it to the user, or send an error message. Each module is then configured from the list of parameters associated to the module name. At last, the **ObservableService** gives all objects that have just been constructed as parameters of the target function (**computeDVCSObservable** here).

The whole sequence is recapitulated in Fig. 8. The XML file dictates the evaluation of a DVCS observable at a kinematic configuration (x_B, t, Q^2, ϕ) . As mentioned, in this example, the observable is $A_{LU}^-(\phi)$, which is a ratio of (combinations) of DVCS cross sections. The kinematic configuration (x_B, t, Q^2, ϕ) is then transferred to the chosen class inherited from **DVCSModule**, say **GV2008Model**. To evaluate a cross section, the parent class requires some values of CFFs, and turns to **DVCSConvCoeffFunctionModule** with the (now restricted) kinematics (ξ, t, Q^2) to evaluate CFFs at LO, with the default conversion⁴ $\xi = x_B/(2 - x_B)$. Evaluating CFFs means computing integrals of GPDs (here **GK11Model**) at $(x, \xi, t, \mu_F, \mu_R)$ with x *a priori* selected by the integration routine, and renormalization and factorization scales chosen by the user as part of his modeling assumptions (*e.g.* $\mu_R^2 = \mu_F^2 = \lambda Q^2$). The kinematic configuration at which the observable should be evaluated has been converted and transmitted from top to bottom. The other way around returns sequentially the evaluation of the GPDs, of the CFFs, of the cross section, and of the considered asymmetry. The final result is stored in a database.

Doing so, the automation file is totally generic: it is independent of the different modules or services. We can use a generic parser, and a generic description of the XML file. It is one more answer to our need of flexibility. The architecture of PARTONS can evolve without any modifications in the parser or in the XML file description.

4.5 Database: storage and transactions

A database is needed for several reasons. We want to keep track of the results of our computations, and once a result is validated, to keep it and not compute it again

³The Registry contains a dictionary that associate a **classId** to a name given as a **string**.

⁴This is the default choice, but other possibilities exist.

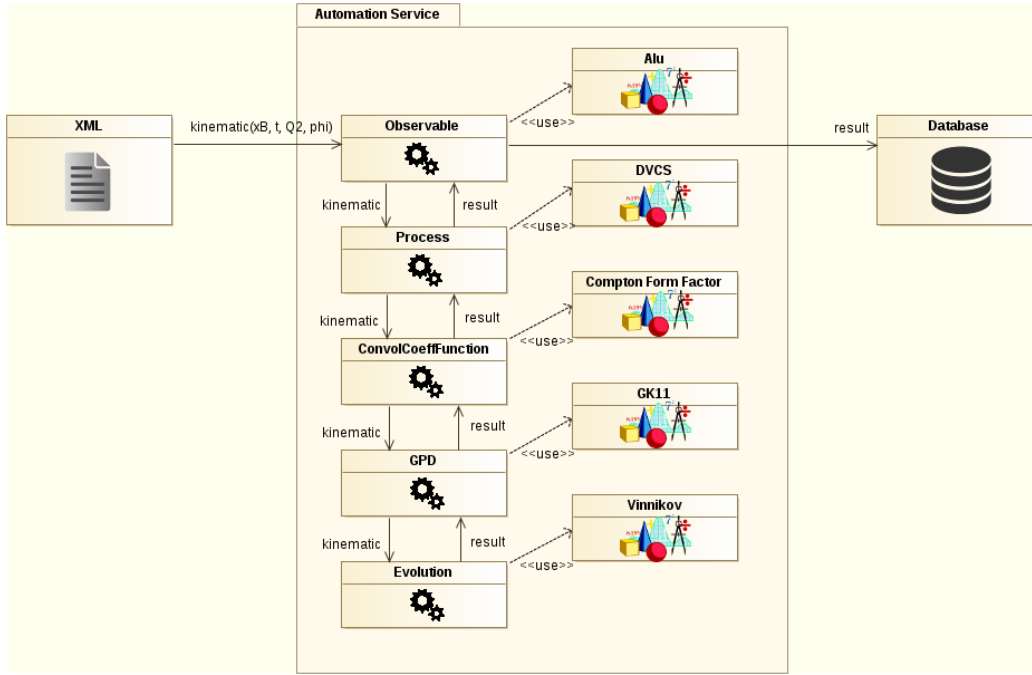


Fig. 8 Principle of automation: computation of a beam spin asymmetry.

anymore. With a related database entry containing the XML file producing the result, it becomes easy to see how something was computed, even if we ask ourselves a long time after. It may also well be that the computational cost of some GPD model is prohibitive, in which case the predictions of this model can be computed separately on a dedicated cluster, stored in the database, and then used for the computation of an observable. The structure of the `GPDModule` is designed so as to make transparent to the user the fact that the GPD values come from a database instead of a direct numerical evaluation. At last, we can also store experimental results in the database, and make systematic comparisons of experimental results and theoretical predictions. Database are optimized to make data selections, in which case kinematic cuts (for example cuts on t/Q^2 to probe the Bjorken limit) are simple and efficient.

In the PARTONS architecture, one layer is dedicated to the transactions with the database. At this level, we should ignore what is the explicit type of database (e.g. a local database on a laptop, or a database on a distant server). The PARTONS code produce data, give them to the *Data Access Object* (DAO) layer, which stores them into the database.

For the sake of simplicity, we discuss the case of the computation of an observable. We store both the value of the observable and the associated kinematics. Thus there are two DAO services, `ObservableKinematicDao` Service and `ObservableResultDaoService`, which

transform the corresponding C++ objects into a collection of simple types (`int`, `double`, `string`, etc.): it is the *serialization* step. The DAO services obey the same pattern as the other services in PARTONS: they receive as inputs and return as results high-level objects instead of simple types. The DAO services then call the necessary DAO objects.

In our example, there are two of them: `ObservableKinematicDao` and `ObservableResultDao`. In this object we can define as many functions as there are requests to make to the database: `insert` (to add a kinematic configuration in the database), `select` (to read a kinematic configuration in the database), `delete` (to suppress a kinematic configuration in the database), etc. Any type of requests can be implemented that way. The following excerpt shows the code underlying the insertion in the database, by the `ObservableKinematicDao` object, of a kinematic configuration starting from simple `double` and `string` variables:

```

1 int ObservableKinematicDao::insert(double xB, double t, double
  Q2,
2     double phi) const {
3     int result = -1;
4     QSqlQuery query(DatabaseManager::getInstance()->
      getProductionDatabase());
5
6     query.prepare(
7         "INSERT INTO observable_kinematic(bin_id, xB, t, Q2,
      phi) VALUES(:bin_id, :xB, :t, :Q2, :phi)");
8
9     query.bindValue(":bin_id", 0);
10    query.bindValue(":xB", xB);
11    query.bindValue(":t", t);
12    query.bindValue(":Q2", Q2);

```

```

13 query.bindValue(":phi", phi);
14
15 if (query.exec()) {
16     result = query.lastInsertId().toInt();
17 } else {
18     error(_func_,
19         Formatter() << query.lastError().text().
20         toString());
21     << "for_sql_query="
22     << query.executedQuery().toString();
23 }
24 query.clear();
25
26 return result;
27 }

```

PARTONS generates the SQL request, which is a simple **string**. This text is interpreted by Qt to replace the dynamical fields (here x, ξ, t, μ_F and μ_R) by their actual value (here **double** variables). The Qt management of connectors⁵ makes possible to send the same SQL request to different databases. The connection of the PARTONS objects, specific to our needs, to the simple types in the database, is done once, and only once, whatever the type of the database is.

The **ObservableKinematicDaoService** performs the same tasks, but this time with the PARTONS **ObservableKinematic** object. This service starts by unfolding the high-level objects into simple types, and run the **insert** function of **GPDKinematicDao**:

```

1 int ObservableKinematicDaoService::insert(
2     const ObservableKinematic &observableKinematic) const {
3     return m_observableKinematicDao.insert(observableKinematic.
4         getXB(),
5         observableKinematic.getT(), observableKinematic.
6         getQ2(),
7         observableKinematic.getPhi());
8 }
9
10 int ObservableKinematicDaoService::getKinematicId(
11     const ObservableKinematic & observableKinematic) const {
12     return m_observableKinematicDao.select(observableKinematic.
13         getXB(),
14         observableKinematic.getT(), observableKinematic.
15         getQ2(),
16         observableKinematic.getPhi());
17 }

```

There are as many DAO classes than there are tables in the database. In that respect, the database structure reflects the architecture of the C++ code.

4.6 PARTONS project: full structure

To summarize, the full architecture of the PARTONS software is summarized on Fig. 9 which displays the layer structure, and the main services.

⁵A connector is a library provided by the editors of the database which permits transactions with a database. This library is written in different languages, *e.g.* C++, Java, Python...

5 Existing modules

The following modules are currently integrated in the PARTONS platform:

GPD The GK model [70–72], the VGG model [7, 73–75], and the GPD models used in the papers by Vinnikov [77] and Moutarde *et al.* [47].

Evolution The Vinnikov code [77].

CFF The convolution of coefficient functions and GPDs evaluated at LO and NLO with adaptive integration routines of Ref. [47] and its extension to the massive quark case [78].

DVCS The set of unpublished analytic expressions of Guichon and Vanderhaeghen used in Refs. [58, 79], and the latest set of expressions [37] by Belitsky and Müller.

Alpha Four-loop perturbative running of the strong coupling constant from PDG [80], and constant value.

This set is by no means limiting. Other modules should be integrated in the platform to allow the systematic differential studies which required the flexible design of PARTONS. All the previous categories should be extended by new modules, either to recover the functionalities of existing codes, or to test brand new development in the integrated chain between models and measurements.

The Vinnikov code has been modified to allow changing the number of active flavors in the code. The number of active flavors N_f in PARTONS appears in four different places with each time a different name:

N_f^{Gen} It is the number of flavors we want to describe in the model: all the flavors hard-coded in the corresponding class, or part of them, or even more of them to radiatively generate flavors. This is the number which specifies the size of the matrix switching from flavor basis to singlet and nonsinglet basis.

N_f^{Evol} It is the number of flavors appearing in the expression of the evolution kernels with the singlet combination defined as the sum over quark flavors q of the combinations $F^{q,(+)}$.

N_f^{Coef} It is the number of flavors appearing in the expression of the coefficient functions for various channels.

N_f^{Alpha} It is the number of flavors appearing in the running of the strong coupling constant. Mentioning it is rather artificial since it is encapsulated in the object describing the running of the strong coupling and cannot be seen outside of this.

Numbers of active flavors are encoded into an **ActiveFlavorModule** that describes piecewise constant functions of the scale μ . These functions take all their values in $\{1, 2, \dots, 6\}$, are increasing with μ and cannot have a

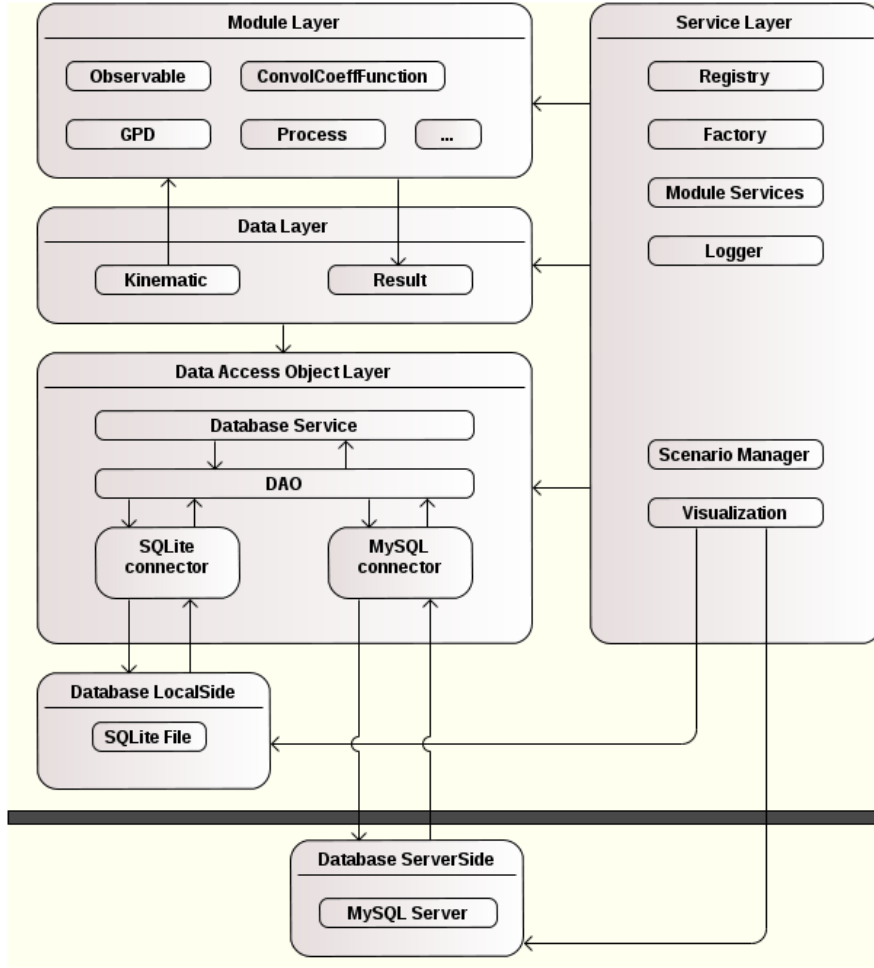


Fig. 9 Full structure of the PARTONS project: layers and services.

jump > 1 at a discontinuity. We will run evolution equations between the scale μ_F^{ref} at which a GPD model is defined, and the "target" scale μ_F . We distinguish two cases:

1. If $\mu_F > \mu_F^{\text{ref}}$, the interval $[\mu_F^{\text{ref}}, \mu_F]$ is decomposed into the number of constant pieces of N_f^{Gen} in the interval $[\mu_F^{\text{ref}}, \mu_F]$. At each threshold a new flavor is added with initial value 0, and a matrix convert GPD combinations from flavor basis to singlet and nonsinglet basis (and vice-versa) at each step to ensure the continuity of GPDs in flavor basis.
2. If $\mu_F < \mu_F^{\text{ref}}$, QCD evolution equations are run downwards, but with a fixed number of active flavors corresponding to the value at μ_F^{ref} .

All these treatments are made in the parent class `GPDEvolutionModule`, and the specific implementation of evolution equations (formulation, kernels, resolution in x -space or in conformal space) is left to daughter classes.

Following our experience in the computation of CFFs [47], we will adopt x -space integration as the default way of solving QCD evolution equations. It should also be mentioned that Gegenbauer polynomials diagonalize evolution equations at LO. This is very useful to test the numerical resolution of evolution equations (if the GPD consists in a single harmonics before evolution, it has to stay so after evolution), but can hardly be used in practice. Belitsky *et al.* [81] reported an oscillating behavior in LO evolution of series of Gegenbauer polynomials, requiring a prohibitively large numbers of terms in the series to recover a smooth function. This is no surprise: all orthogonal polynomials have simple zeros in the interval where they are defined. Roughly speaking, the n^{th} -harmonic will oscillate around n zeros, with terms going like x^n with $|x| \leq 1$ multiplied by coefficients with a factorial growth. All these reasons suggest that the actual numerical evaluation will be delicate, and it is indeed our experience with simple models on Mathematica that extra care has to be

given to the computation as soon as $n \gtrsim 40$. Note that this problem is less severe in conformal space [81–85] because the hypergeometric functions used in the analytic continuation of Gegenbauer polynomials vanish fast off the real axis. But analytically continuing a GPD model numerically defined by a finite number of conformal moments is a difficult task: the solution (if it exists) is not unique, and may not depend continuously on the known conformal moments.

6 Examples

6.1 Computation of a GPD value without automation

The following example shows how to compute a GPD without automation. Almost each line of the code of the function `gpdExample()` corresponds to a physical hypothesis, but the user still has to explicitly deal with pointers.

```

1 #include <src/beans/gpd/GPDKinematic.h>
2 #include <src/beans/gpd/GPDResult.h>
3 #include <src/beans/gpd/GPDType.h>
4 #include <src/database/service/GPDResultDaoService.h>
5 #include <src/modules/gpd/GK11Model.h>
6 #include <src/ModuleObjectFactory.h>
7 #include <src/Partons.h>
8 #include <src/services/GPDService.h>
9 #include <src/ServiceObjectRegistry.h>
10 #include <exception>
11 #include <iostream>
12 #include <string>
13 #include <vector>
14
15 void gpdExample() {
16     std::cerr << "#####_gpdExample()_#####" << std::endl;
17
18     //////////////////////////////////////////
19     // HERA kinematics                      //
20     // xB = 0.001 or xi = 0.00050025      //
21     // t = -0.3 GeV2                      //
22     // Mu2 = 8. GeV2                      //
23     //////////////////////////////////////////
24
25     // Retrieve GPD service
26     GPDService* pGPDService = ServiceObjectRegistry::
getGPDService();
27
28     // Load GPD module with the BaseModuleFactory
29     GPDModule* pGK11Model = ModuleObjectFactory::
newGPDModule(
30         GK11Model::classId);
31
32     // Create a GPDKinematic(x, xi, t, MuF, MuR) to compute
33     GPDKinematic gpdKinematic(0.1, 0.00050025, -0.3, 8., 8.);
34
35     // Compute data and store results
36     GPDResult gpdResult = pGPDService->
computeGPDModelRestrictedByGPDType(
37         gpdKinematic, pGK11Model, GPDType::ALL);
38
39     // Print results
40     std::cout << gpdResult.toString() << std::endl;
41
42     delete pGK11Model;
43     pGK11Model = 0;
44 }
45
46 int main(int argc, char** argv) {
47     try {
48
49         // Init PARTONS application

```

```

50     Partons* pPartons = Partons::getInstance();
51     pPartons->init(argv);
52
53     // ##### GPD #####
54     gpdExample();
55
56     // Stop PARTONS application
57     pPartons->close();
58
59     } catch (std::exception &e) {
60         std::cerr << e.what() << std::endl;
61     }
62
63     return 0;
64 }

```

6.2 Automated computation of a GPD value

The following example shows how to compute a GPD with automation. Each line of the XML file corresponds to a physical hypothesis, and the user does not have to explicitly deal with pointers anymore.

```

1 <?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
2
3 <scenario id="01" date="" description="Example of
one GPD model
(GK11) without evolution">
4     <!-- Select type of computation -->
5     <task service="GPDService" method="computeGPDModel" >
6
7
8         <!-- Specify kinematics -->
9         <GPDKinematic>
10             <param name="x" value="0.1" />
11             <param name="xi" value="0.00050025" />
12         </GPDKinematic>
13
14         <param name="t" value="-0.3" />
15         <param name="MuF2" value="8" />
16         <param name="MuR2" value="8" />
17     </task>
18
19     <GPDModule>
20         <param name="id" value="GK11Model" />
21     </GPDModule>
22 </scenario>

```

6.3 Automated computation of a beam spin asymmetry

The following example shows how to compute the beam spin asymmetry $A_{LU}^-(\phi)$ defined in Eq. (28) on a kinematic configuration typical of Jefferson Lab upgraded at 12 GeV: $x_B \simeq 1/3$ ($\xi \simeq 0.2$), $t = -0.2$ GeV², $\mu_F^2 = \mu_R^2 = 4$ GeV², and with a beam energy $E_{\text{Lab}} = 11$ GeV. This example is close to the one discussed in Sec. 4.4, but this time the code is executed with a list of values of ϕ ranging between 0 and 360 degrees. This is indicated by the method `computeManyKinematicOneModel`. The list of kinematic configurations is provided in a file as indicated between the markups `<ObservableKinematic>` and `</ObservableKinematic>`, and is described as simple text:

0.333|-0.2|4.0|0.0
 0.333|-0.2|4.0|-3.6
 0.333|-0.2|4.0|-7.2
 0.333|-0.2|4.0|-10.8
 0.333|-0.2|4.0|-14.4
 ...

where we can see, on each line, from left to right: x_B , t (in GeV^2), Q^2 (in GeV^2), and ϕ (in degrees). The resulting curve is shown in Fig. 10.

```

1 <?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
2
3 <scenario id="05" date="" description="Comparison_of_Al_u_with_
  LO_and_NLO.">
4
5   <task service="ObservableService" method="
  computeManyKinematicOneModel" storeInDB="0">
6     <ScaleModule>
7       <param name="id" value="Q2Multiplier"
8     />
9     <param name="lambda" value="1" />
10    </ScaleModule>
11
12    <XiConverterModule>
13      <param name="id" value="XBToXi" />
14    </XiConverterModule>
15
16    <ObservableKinematic>
17      <param name="file"
18        value="/home/debian/workspace/
  PARTONS/data/phi.dat" />
19    </ObservableKinematic>
20
21    <Observable>
22      <param name="id" value="Alu" />
23    </Observable>
24
25    <DVCSModule>
26      <param name="id" value="BMJ2012Model"
27    />
28      <param name="beam_energy" value="11" />
29    </DVCSModule>
30
31    <DVCSConvCoeffFunctionModule>
32      <param name="id" value="DVCSCoeffModel"
33    />
34      <param name="qcd_order_type" value="
  LO" />
35    </DVCSConvCoeffFunctionModule>
36
37    <GPDMModule>
38      <param name="id" value="GK11Model" />
39    </GPDMModule>
40  </task>
41 </scenario>

```

7 Conclusions

In the last twenty years we have witnessed an intense theoretical and experimental activity in the field of exclusive processes described in terms of Generalized Parton Distributions. It is also a crucial part of the forthcoming experiments at Jefferson Lab, COMPASS and in the future at EIC. The amount and quality of the expected data, together with a richness and versatility of theoretical approaches to its description, calls for a flexible, stable and accurate software platform that will allow for systematic phenomenological studies. In this

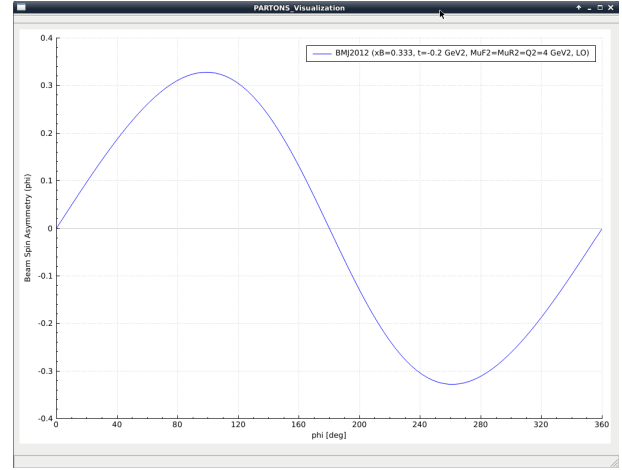


Fig. 10 Beam spin asymmetry $A_{LU}^{-}(\phi)$ for $x_B \simeq 1/3$ ($\xi \simeq 0.2$), $t = -0.2 \text{ GeV}^2$, $Q^2 = \mu_F^2 = \mu_R^2 = 4 \text{ GeV}^2$, and $E_{\text{Lab}} = 11 \text{ GeV}$. CFFs are evaluated at LO.

paper we have described such a platform, called PARTONS, and how it addresses the most important tasks: automation, modularity, non-regression, data storage and visualization. We have also provided a simple example of the XML *scenario* files illustrating automated calculations of physical observables.

Since its inception three years ago, the PARTONS framework has expanded rapidly with the addition of new core developers. We intend, in the mid- to long-term future, to complement PARTONS with new theoretical developments, new computing techniques and other exclusive processes. To achieve this, we expect that more physicists will join the development team to integrate new modules and benefit from our integrated chain, relating theory to experimental observables. PARTONS should become the *de facto* software framework for the GPD analysis of the next-generation exclusive data.

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